

# A Mathematical Model Of Microplastic Spreading Into Fish Digestive Based On Abiotic Factor

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## Abstrak

Dalam penelitian ini, kami mengamati ikan dari tujuh lokasi sungai yang berbeda di Yogyakarta dengan mengevaluasi berat pencernaannya. Penebaran mikroplastik pada saluran pencernaan ikan dilakukan berdasarkan faktor abiotik seperti suhu sungai, keasaman, dan butiran mikroplastik aliran sungai untuk dibawa ke saluran pencernaan ikan. Kadar mikroplastik dalam tubuh ikan dapat dideskripsikan secara matematis menggunakan persamaan diferensial. Kami membangun model berdasarkan diagram aliran hubungan antara masing-masing variabel. Jadi kami memiliki sistem diferensial sebagai model. Pada langkah selanjutnya kami menganalisis model secara analitis. Untuk menunjukkan keakuratan model, kami membuat simulasi menggunakan simulasi data ke sistem dan membandingkannya dengan hasil komputasi menggunakan data observasi. Di akhir penelitian, kami memberikan justifikasi untuk faktor abiotik yang paling berpengaruh terhadap penyebaran mikroplastik.

**Kata Kunci:** Model Matematika; Mikroplastik; Persamaan Diferensial

## Abstract

In this research, we observe the fish from seven different river location on Yogyakarta by evaluating its digestive weight. We investigate the microplastics spreading on fish digestive based on the abiotic factor such as river temperature, acidity, and river flow microplastics granules to be carried into the digestive tract of the fish. The rate of microplastics in the fish body can be describe mathematically using differential equation. We build a model based on the diagram flow of the relationship between each variabels. Thus we have a differential system as the model. In the next step we analyze the model analytically. To show the accuracy of the model, we make a simulation using data simulation to the system and we compare it with the computing results using observation data. At the end of our research, we give a justification for the most influential abiotic factor for microplastic sreading.

**Keywords:** Differential equation; Microplastic; Mathematical model

## Introduction

Waste is one of the biggest problem on this country and plastic waste that dominates the most. As we already know that plastic is not biodegradable. It only breaks up into smaller pieces. Those particles are very dangerous for the environment, especially when it washed away in the water. It can be consumed by the organism lived in it such as the fish. It will be more dangerous if the fish are consumed by humans since microplastics cannot be digest by the human body.

This issue has been widely studied among researcher. In [1] investigate microplastic in the digestive tracks of 64 Japanese anchovy. It shows that there was microplastic detected on 77% fish population with various types of plastics. In the other research, [2] study the presence of microplastics in the digestive tract on some fish at Pantai Baron, Yogyakarta, Indonesia. They worked by extracting fish digestive track with 10% KOH. They summarizes that microplastics identified on fish digest track was because the nature of microplastics that is lightweight and floats on the river. While [3] and [4] were documenting on the existence of microplastics in wild fish larvae which taken from three different side and the microplastic density on fish digest tract is significantly increasing. In the next literature, they reports that

plastic ingestion in various fish from coastal and offshore sites from 128 species detected, starts with microplastic until macroplastic with a different type of plastic.

In this paper, we investigate three factor effecting microplastic transported to fish digest track. Based on the hypothesis, the cause of microplastic on fish digestive track are water temperature, acidity and water flow intensity. It also supported by [5] temperature and chemical digestion effecting to the amount of microplastic that entering to fish digestive. Those three factor will be examine in our discussion. We use a mathematical approach to build the model and analyze the process. The model will be constructed using differential equation. Some of them discuss mathematical models in fish.

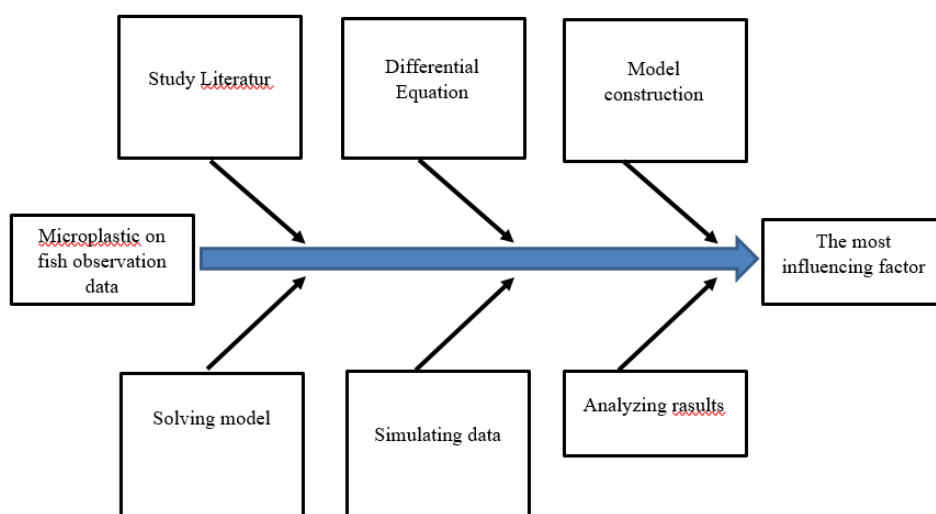
There are many researches on differential equation which is applied to real problem, for example research [6] describing fish schooling process using stochastic diferential equation model which give a unique local solution. At the other research, [7] developed a model for spontaneous fish movement to analyze Kahlia mugil fish which freely swim in a tank. It also comparing the process with a standart persistent random walk. In [8] discussing a model to describe a water flow in and on fish mouth. It consisting an expanding and compressing rational movement using partial deferential equation, specifically using Naïve-Stokes equation.

In this research, we work with differential equation to analyze the factors causing microplastic transfer into fish digestive. We observe the relation for each factors to the sum of microplastic consumed by fish on Yogyakarta river. Further, we give a numerical simulation to show the most potentials factor to this problem. The results will be compared with the real data obtained from the observation.

**Research Method**

In this section we give the method used for this research. At the beginning, we have an observational data including the number of microplastic in fish digestive, water temperature, acidity of the rivers in Yogyakarta, Indonesia. This data is analyze using mathematical tools of differential equation. Hance, we study some literature that discuss about differential equation, fish modeling process.

Further, we choose the appropriate differential equation to fit those problems. Then, a differential model for fish digestive track can be construct. Next, we simulate the solution of this model using a simulation data and observation data. Such that, we can compare the results for both simulation and observation. Finally we have the most influencing factor for transporting microplastic from the river into fish digestive track. The overall method can be seen on Figure 1 below.



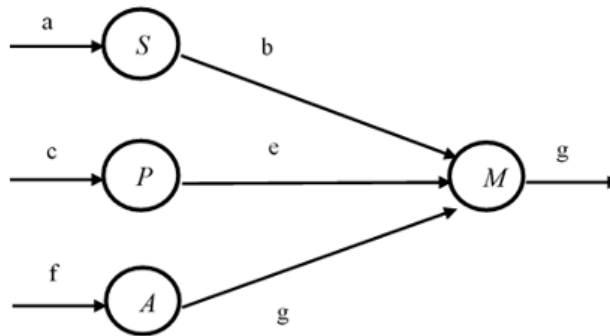
**Figure 1** Methods of the research

**Results and Discussion**

In this section, we construct a mathematical model to modelling the factor causing microplastic transporting to fish digestive track. We construct four compartment including to three factors effecting microplastic granule inside fish digestive. In this model, we restricting the factors involved i.e. water temperature, acidity and water flow of the rivers. The compartment representing the percentage of microplastic on fish digestive effected by water temperature is given by  $S(t)$ , the percentage of microplastic on fish digestive effected by water acidity is given by  $P(t)$ , the percentage of microplastic on fish digestive effected by water flow of the rivers is given by  $A(t)$  and the percentage of the number of microplastic on fish digestive is  $M(t)$ . The number of population in this system equals to the sum of the total individual on each compartment

$$N(t) = S(t) + P(t) + A(t) + M(t) \tag{1}$$

Each variable contributes to the number of microplastic transport to fish digestive which can be show on the compartment diagram below.



**Figure 2** Compartment diagram of external factor causing microplastic entering fish digestive

In this case, the number of microplastic increases as increased naturally occurring microplastics in ecosystems such as regular garbage disposal to the river and microplastics carried by water flows from the upstream before. Further, the number of microplastics in the fish body which influenced by temperature, pH and river flow velocity, each of which is imposed on the multiplier of the number of microplastics, namely the parameters . Those parameter can be describe on Table 1.

**Table 1** Parameter used in this model

Paramete r	Description.
$a, c, f$	Percentage rate of microplastic increasing naturally
$b$	The percentage rate of increase in the number of microplastics due to river water temperature
$e$	The percentage rate of increase in the number of microplastics is due to the acidity of the river water
$g$	The percentage rate of increase in the number of microplastics due to the velocity of river water flow
$i$	Percentage rate of total increase in the number of microplastics

The above model can be described in the following equation mathematically.

$$\left\{ \begin{array}{l} \frac{dS}{dt} = a - bS \quad (a) \\ \frac{dP}{dt} = c - eP \quad (b) \\ \frac{dA}{dt} = f - gA \quad (c) \\ \frac{dM}{dt} = bS + eP + gA - iM \quad (d) \end{array} \right. \quad (2)$$

with the given initial state as below

$$S(0) = S_0 \geq 0, P(0) = P_0 \geq 0,$$

$$A(0) = A_0 \geq 0, M(0) = M_0 \geq 0$$

and

$$a, b, c, e, f, g > 0.$$

To show that the solution of Equation 2.a-2.d is a non-negative solution if a non-negative initial condition given to the system, we give the following theorem.

**Teorema 1:** Given a set of active and positive invariant  $\Omega = \left\{ (S, P, A, M) \in R^4 : S \leq \frac{a}{b}, P \leq \frac{c}{e}, A \leq \frac{f}{g} \right\}$  for

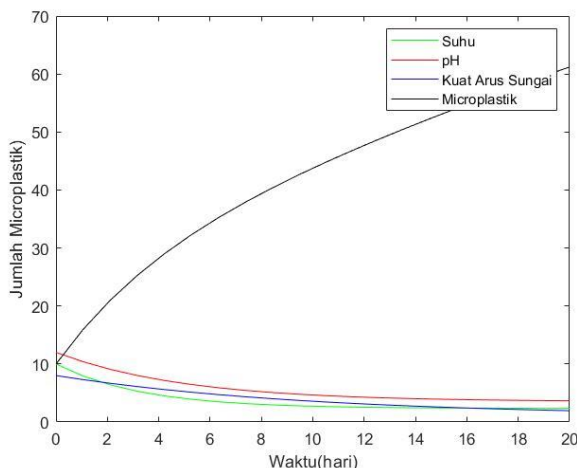
(2), then the solution is non-negative.

In this research, a numerical simulation test performed using data simulation and data observation. First, we give a simulation data to get an initial analysis. The simulation data can be shown on Table 2 below.

**Table 2** Simulation Data for each parameter

Paramete r	Description
<i>a</i>	0.7
<i>b</i>	0.3
<i>c</i>	0.7
<i>e</i>	0.2
<i>f</i>	0.1
<i>g</i>	0.1
<i>i</i>	0.001

This simulation parameter data is still in accordance with the percentage of the amount of microplastic present in river water based on the influence factor. By performing a numerical test on 2, we get a simulation result as follows.



**Figure 3** The graph of numerical simulation of the number of microplastics in the digestive tract of fish.

Through this numerical simulation, it can be seen that the factors of temperature, pH and river flow strength do not significantly affect the number of microplastics in the digestive tract. The number of microplastics is based on temperature, pH and asymptotic water current at intervals of 2.5-5.5 units. However, the accumulation of each of these factors causes a cumulative increase in the number of microplastics in the digestive tract of fish as shown in the black graph. By simulation, the estimated number of microplastics in the digestive tract of fish can be written in Table 3 below.

**Table 3** Prediction of the number of microplastics in the digestive tract of fish based on simulation data

Time	Microplastic Prediction			
	Temperature	Acidity	Water Flow	Microplastic
0	10	12	8	10
2	9	6.5	6.5	20.8
6	3.4	4.7	5.9	35.2
12	3.07	3.69	4.83	47.08
16	2.77	4.16	2.77	54.69
20	2.77	4.16	2.77	61.22

For the second, we show the numerical test using observation data. The observation data can be describe on the Table 4 below.

**Table 4** Observation Data for each parameter

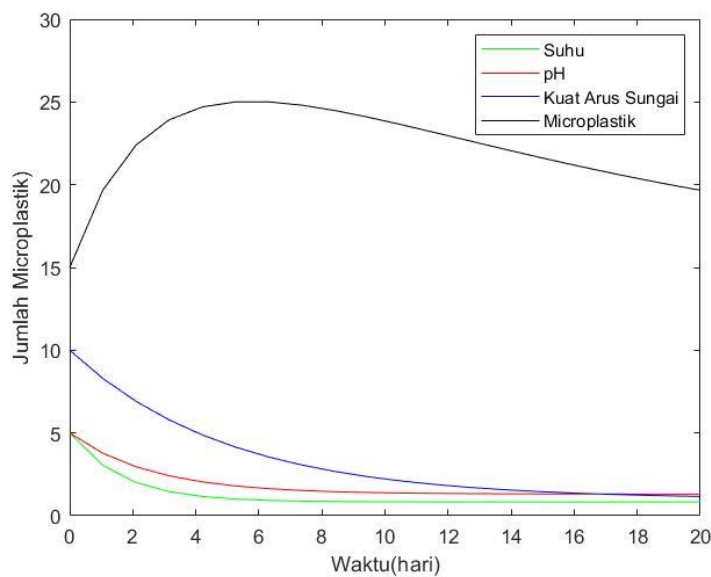
Parameter	Description
<i>a</i>	0.5
<i>b</i>	0.602
<i>c</i>	0.5
<i>e</i>	0.383
<i>f</i>	0.2
<i>g</i>	0.2
<i>i</i>	0.008

In the simulation of the observation data, the prediction of the number of microplastics is obtained which is not much different from the simulation data which influenced by temperature, pH and river flow strength. Meanwhile, the total accumulative amount of microplastics in the digestive tract of fish has increased until the third to the twelfth day. These results can be explained in the Table 5.

**Table 5** Prediction of the number of microplastics in the digestive tract of fish based on observation data

Time	Microplastic Prediction			
	Temperature	Acidity	Water Flow	Microplastic
0	5	5	10	15
2	2	2.95	6.09	22.39
6	0.62	1.33	3.24	24.7
12	0.62	1.33	2.38	24.65
16	0.62	1.33	1.38	21.27
20	0.62	1.33	1.35	19.66

Visually, the prediction of the number of microplastics in the digestive tract caused by external factors based on observational data can be clearly illustrated in Figure 4.



**Figure 3** The graph of the observation data simulation of the number of microplastics in the digestive tract of fish.

**Conclusion**

In this study, a mathematical model of microplastic acceptance was formed in the digestive tract of fish from six rivers in Yogyakarta. There are four compartments involved, namely temperature, acidity, strong river flow and the amount of microplastic which is considered to be a factor causing the entry of microplastics into the digestive tract of fish. The numerical simulation given in this study is given simulation data as well as simulations with real data obtained from the field. Based on the results of numerical simulations and real data simulations, it is concluded that the factors of temperature, pH and river flow strength do not play a significant role in the process of entering microplastics in fish bodies. This is indicated by the increase in the cumulative number of microplastics in fish. This also shows that there are other factors that significantly influence the entry of microplastics into the digestive tract of fish. Further investigation regarding the factors that greatly influence the number of microplastics that enter the digestive tract of fish will be carried out in future studies.

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